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In this letter the paper [R. Aquilano, M. Castagnino, Mod. Phys. Lett A, **11**, 755 (1996)] is improved by considering that the main source of entropy production are the stars photospheres.

I. INTRODUCTION.

In papers [1] [1] and [2] two of us reported a rough coincidence between the time where the minimum of the entropy gap $\Delta S = S_{act} - S_{\max}$ [3], takes place and the time where all the stars will exhaust their fuel. The rough nature of this result was explained saying that the calculation was done using a model extremely naive and simplified: a homogeneous universe [2]. In fact in the real universe nuclear reactions (that were considered as the main source of entropy) take place within the stars, that can only be properly considered in an inhomogeneous geometry. So we expend some time trying to generalize our model to these type of geometries and computing the entropy of the galaxies and stars in the condensation period, after decoupling time, in order to make the coincidence precise [3]. It was soon clear to us that the latter entropy can be neglected with respect to the amount of entropy produced within the stars and therefore the mechanism that produced the inhomogeneity is not so important for our problem. Stars are formed when a sufficiently large mass of interstellar gas is compressed into a small enough volume, so its force of self-gravitation becomes sufficiently great to cause gravitational collapse. The instability makes the cloud to reduce its size quickly, the temperature rises and pressure forces begin to restrict the collapse until hydrostatic equilibrium is obtained. In this process, gravitational energy is converted in kinetic energy and radiation [4]. In the classical model of star formation the time of contraction is the Kelvin-Helmholtz time [5]:

$$t_{KH} = \frac{GM^2}{RL} \tag{1}$$

where M is the mass, R is the radius and L is the luminosity of the star. For a star like the sun this time is $3\ 10^7$ years [4] [5]. In more recent models of star formation, the relevant time scale is the much shorter free-fall time [5]:

$$t_{ff} = (\rho G)^{-1/2} = 7 \cdot 10^5 \left(\frac{n}{10^4}\right)^{-1/2} years \tag{2}$$

where ρ and n are the cloud mass density and the cloud number density respectively.

In any case, it takes less than 1% of the lifetime ($\cong Gyr$) to the star to form. In the contraction to make the star, the energy radiated is a part of the gravitational energy and the nuclear reactions are not important. The binding gravitational energy is negligible compared with the energy liberated through nuclear reactions when the star is already formed [4]. As a consequence we can neglect the entropy production in the phase of contraction that makes the star.

So, in order to improve our rough result, we were forced to change the approach.

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[1] It is a good occasion to correct some errata in the abstract of paper [1].
-In line 2: it reads "quantitative", it must read "qualitative".
-In line 3: it reads "entropy", it must read "entropy gap".
-In line 4: it reads "qualitative", it must read "quantitative".

[2] Besides that the higher order terms in eq. (3) of ref. [1] were neglected; perhaps they may be important for finite times.

[3] In fact, inhomogeneity must be considered, but in a different way, as we will see below.

Let us review the main formulae. The entropy gap was the conditional entropy of $\rho(t)$, the state of the universe at time t with respect to the equilibrium state at that time $\rho_*(t)$:

$$\Delta S = -tr[\rho \log(\rho_*^{-1}\rho)] \tag{3}$$

where

$$\rho(t) = \rho_*(t) + \rho_1 \exp(-\gamma t) + O[(\gamma t)^{-1}] \tag{4}$$

and

$$\rho_*(\omega) = ZT^{-3}\frac{1}{\exp(\frac{\omega}{T} - 1)} \tag{5}$$

ρ_1 was a phenomenological coefficient constant in time, the time variation of the main irreversible process was $\exp(-\gamma t)$ so γ^{-1} was the characteristic time of this process, T was the temperature of the universe, $T \sim a^{-1}$ where a is the radius or scale of the universe, and Z is a normalization constant.

Then we could approximate ΔS as

$$\Delta S = -CT^3 \exp(-\gamma t) \exp(\frac{\omega_1}{T}) \tag{6}$$

where ω_1 was the characteristic energy where ρ_1 is peaked.

The time where the minimum of ΔS is located was:

$$t_{cr} \approx t_0 \left(\frac{2}{3}\frac{\omega_1}{T_0}\frac{t_{NR}}{t_0}\right)^3 \tag{7}$$

We selected numerical values for the parameters: $\omega_1 = T_{NR}$, the temperature of the nuclear reactions within the stars (that was used in paper [1] as the main source of entropy), $t_{NR} = \gamma^{-1}$ the characteristic time of these nuclear reactions, t_0 the age of the universe, and T_0, the cosmic micro-wave background temperature.

T_{NR} and t_{NR} were chosen between the following values [6]:

$$T_{NR} = 10^6 \ to \ 10^8 \ K \tag{8}$$

$$t_{NR} = 10^6 \ to \ 10^9 \ years$$

while for t_0 and T_0 we can take:

$$t_0 = 1.5 \times 10^{10} \ years \tag{9}$$

$$T_0 = 3K$$

In order to obtain a reasonable result we chose (with no explanation) the lower bounds for T_{NR} and t_{NR} and for t_{cr} we obtained :

$$t_{cr} \preceq 10^4 t_0 \cong 10^{14} years \tag{10}$$

concluding that the order of magnitude of t_{cr} was a realistic one. In fact, $10^4 t_0 \approx 1.5 \times 10^{14} years$ after the big-bang all the stars will exhaust their fuel [7][4], so it is reasonable that this time would be of the same order than the one where the entropy gap stops its decreasing and begins to grow [9]. But the choice of the lower bound in eqs. (8) was not explained and only the inhomogeneity was argued as above.

[4] After the publication of paper [1], in 1996, new data about this time appear in [8].

II. THE PHOTOSPHERE AS THE MAIN UNSTABLE SYSTEM.

Now we have reconsidered the problem and we conclude that, even if nuclear reactions within the stars are the main source of entropy, the parameters T_{NR} and t_{NR} are not the good ones to define the behavior of the term $\exp(-\gamma t)\rho_1$ of equation (3)((3) of paper [1]), since they do not correspond to the main unstable system that we must consider [5]. In fact the main production of entropy in a star is not located in its core, where the temperature is almost constant (and equal to T_{NR}), but in the photosphere where the star radiates. The energy radiated from the surface of the star is produced in the interior by fusion of light nuclei into heavier nuclei. Most stellar structures are essentially static, so the power radiated is supplied at the same rate by these exothermic nuclear reactions that take place near the center of the star [4]. Once the star is formed, it settles into a termally stable state where all the nuclear energy is radiated at the surface and the rate of internal entropy change is extremely low [5].

We can decompose the whole star in two branch systems [3], as explained in paper [1] (or in section VII of paper [2]), where a chain of branch systems was introduced. We have two branch systems to study: the core and the photosphere. The core gives energy to the photosphere and in turn the photosphere diffuses this energy to the surroundings of the star, namely in the bath of microwave radiation at temperature T_0 .In this way, we have two sources of entropy production: the radiation of energy at the surface of the star and the change of composition inside the star (as time passes we have more helium and less hydrogen). Since the core of a star is near thermodynamic equilibrium, we neglect the second and we concentrate on the first: the radiation from the surface of the star (related with the difference between the star and the background temperatures). So the temperature of the photosphere and not the one of the core must be introduced in our formula. This is also the case for the lifetime. We must take the lifetime of the photosphere not the one of the nuclear reactions. Thus it is better to consider the photosphere as the unstable system that defines the term $\exp(-\gamma t)\rho_1$ of equation (4). So we must change T_{NR} and t_{NR} by T_P, the temperature of the photosphere and t_S the characteristic lifetime of the star respectively. Then we must change eq. (7) to:

$$t_{cr} \approx t_0 \left(\frac{2}{3} \frac{T_P}{T_0} \frac{t_S}{t_0} \right)^3 \tag{11}$$

Considering that the mean mass of stars is $0.64 M_\odot$ with surface temperature $4.6 \cdot 10^3 K$ and lifetime $3.8 \cdot 10^{10} years$ (see appendix) we obtain:

$$t_{cr} \cong 10^{10} t_0 \cong 10^{20} years \tag{12}$$

but now the computation was not done using an arbitrary choice of the lower bound in some data, but using the first meaningful figure [6] in all the data [7].

III. THE STELLIFEROUS AND THE DEGENERATED ERAS.

Let us now compare this result with those of paper [11], where the future history of the universe is analyzed. $t_{cr} \cong 10^{10} t_0 = 10^{20} years$ is placed after the end of the "stelliferous era" ($10^6 < t < 10^{14} years$), where most of the energy generated in the universe arises from nuclear processes in the conventional star evolution, and at the beginning of the "degenerated era" ($10^{15} < t < 10^{37} years$), where most of the (baryonic) mass of the universe is locked up in degenerated stellar objects: brown dwarfs, white dwarfs, and neutron stars. In this era energy is generated through proton decay and particle annihilation.

So $t_{cr} \cong 10^{20} years$ is not a bad place for the minimum of the entropy gap, since it is bigger than the end of conventional star formation ($10^{14} years$), it is also bigger than the typical time of star formation via brown dwarfs collision ($10^{16} years$), and it is of the order of the time corresponding to stellar evaporation from galaxies ($10^{19} years$). Namely a time where we can consider that the main mechanisms of formation of stars are ended while the beginning

[5]In fact, to take the parameters T_{NR} and t_{NR} corresponds to take an homogenous gas model that fills the universe where, nuclear reactions take place in this homogenous gas, while in real universe these reactions take place within the star in a quite inhomogeneous scenario.

[6]In paper [10], using the photosphere data, but just orders of magnitude, we have again obtained (10).

[7]We have obtained a larger result than (10), but remember that the latter was obtained just by taking the lower bound in eqs. (8). If we would choose the mean values in these equations we would obtain a larger result than the one of eq. (12), without any significance.

of the evaporation process has just started. This is the best place for t_{cr}, that must be the frontier between the growing order period (formation of structures with decaying entropy [9]) and the diminishing order period (decaying of the structures with growing entropy). Tacking into account the great uncertainty of all cosmological data we can say that t_{cr} is located in the edge between the stelliferous normal era, where we are living and which is dominated by the formation of structures, and the future degenerated era, full of, by now strange, objects, where the growing of disorder will begin. So even if eq. (11) is the result of a very simple model it gives a very reasonable value.

IV. CONCLUSION.

By choosing a more realistic model as the main source of entropy production, the photosphere of the stars, we have obtained a reasonable value for the time where the minimum of the entropy gap is reached. This is the frontier of the formation and destruction of structure periods and therefore one of the most important moments of the future history of the Universe.

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VI. APPENDIX

The stars have masses in the interval [12], [13]:

$$0.1 \leq m = \frac{M_*}{M_\odot} \leq 100 \tag{13}$$

where M_* is the mass of the star and $M_\odot$ is the mass of the Sun.

The mean mass of stars can be calculated using the initial mass function (IMF) $\xi(m)$ defined by [13]:

$$dN = \xi(m) \cdot dm \tag{14}$$

where dN is the number of stars with masses between m and $m + dm$

We will use as IMF [11]:

$$\frac{dN}{d(\ln m)} = \psi(\ln m) = \exp\left[A - \frac{1}{2 < \sigma >^2} \ln^2\left(\frac{m}{m_c}\right)\right] \tag{15}$$

where $< \sigma > \cong 1.57$, $m_c \cong 0.1$ and A is a constant that sets the overall normalization of the distribution (which is not important because it cancels in the calculation of the mean mass of stars). This form of the IMF is consistent with observations (see [14]).As

$$\xi(m) = \psi(\ln m) \cdot \frac{d(\ln m)}{dm} = \frac{1}{m}\psi(\ln m) = \frac{1}{m}\exp\left[A - \frac{1}{2 < \sigma >^2} \ln^2\left(\frac{m}{m_c}\right)\right] \tag{16}$$

then the mean mass of stars is:

$$< m > = \frac{\int_{0.1}^{100} m \cdot \xi(m) \cdot dm}{\int_{0.1}^{100} \xi(m) \cdot dm} = 0.64 \tag{17}$$

Let us calculate the effective surface temperature of a star with mass $m = 0.64$. The Luminosity of a star is given by the Stefan- Boltzmann law:

$$L = 4\pi R^2 \sigma T_p^4 \tag{18}$$

where R is the radius of the star, σ is the Stefan- Boltzmann constant and T_p is the effective temperature of the surface of the star. Then

$$\frac{L}{L_\odot}=\left(\frac{R}{R_\odot}\right)^2\left(\frac{T_p}{T_\odot}\right)^4 \tag{19}$$

where $L_\odot$, $R_\odot$,$T_\odot$ are, respectively, the luminosity, the radius and the surface temperature of the Sun.
The radius and luminosity for stars in the main sequence, as a function of mass are (see [15]):

$$\frac{R}{R_\odot}\cong\left(\frac{M_*}{M_\odot}\right)^\beta=m^\beta \tag{20}$$

$$\frac{L}{L_\odot}\cong\left(\frac{M_*}{M_\odot}\right)^\eta=m^\eta \tag{21}$$

with $\beta\cong 0.6$ for stars of low mass and $\eta\cong 3.2$ .So we have

$$T_P\cong T_\odot\left(\frac{M_*}{M_\odot}\right)^{\frac{\eta-2\beta}{4}}\cong T_\odot\cdot m^{0.5} \tag{22}$$

Using that $T_\odot\cong 5780K$ and $m=0.64$ we obtain

$$T_P\cong 4.6\cdot 10^3K \tag{23}$$

The lifetime of a star can be calculated by the equation(see [11]):

$$t_s=10^{10}\left(\frac{M_*}{M_\odot}\right)^{-\alpha} years=10^{10}m^{-\alpha}years \tag{24}$$

where $\alpha\cong 3-4$ for stars of low mass. Taking $\alpha=3$ we have:

$$t_s\cong 3.8\cdot 10^{10}years \tag{25}$$

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